Beam-to-Instrument Transformation

The vector of beam velocities is transformed to the vector of velocity components in a coordinate system fixed to the instrument (with error velocity appended) through multiplication by the instrument transformation matrix. The instrument coordinate system is described in WorkHorse Command and Data Output Formats Documentation. The instrument transformation matrix is displayed by the PS3 command. Unless the ADCP has been calibrated to compensate for small beam misalignments, the instrument transformation matrix has the following nominal value:

component	beam 1	beam 2	beam 3	beam 4
X	c*a	-c*a	0	0
Y	0	0	-c*a	c*a
Z	b	b	b	b
e	d	d	-d	-d

Where: $c = +1$ for a convex transducer head, -1 for concave

$a = 1/[2\sin(\theta)] = 1.4619$ for $\theta = 20°$, 1.0000 for $\theta = 30°$

$b = 1/[4\cos(\theta)] = 0.2660$ for $\theta = 20°$, 0.2887 for $\theta = 30°$

$d = a/\sqrt{2} = 1.0337$ for $\theta = 20°$, 0.7071 for $\theta = 30°$

The first three rows are the generalized inverse of the beam directional matrix representing the components of each beam in the instrument coordinate system (see PS3 command for WorkHorse systems – Contact TRDI Field Service for other systems). The last row representing the error velocity is orthogonal to the other three rows and has been normalized so that its magnitude (root-mean-square) matches the mean of the magnitudes of the first two rows. This normalization has been chosen so that in horizontally homogeneous flows, the variance of the error velocity will indicate the portion of the variance of each of the nominally-horizontal components (X and Y) attributable to instrument noise (short-term error).

The above table is equivalent to the matrix equation below:

$$\begin{bmatrix} X \\ Y \\ Z \\ e \end{bmatrix} = \begin{bmatrix} b_1 ca - b_2 ca \\ b_4 ca - b_3 ca \\ b(b_1 + b_2 + b_3 + b_4) \\ d(b_1 + b_2 - b_3 - b_4) \end{bmatrix} \quad (1)$$

The velocity will be left in instrument coordinates if the EX01xxx command is selected. Usually, an additional rotation is desired, which is accomplished by multiplying the instrument transformation matrix on the left by the rotation matrix M described in Coordinate Transformation Booklet before using it to transform the velocity components of each depth cell and the bottom track velocity.

Three-Beam Solutions

If exactly one beam has been marked bad in the screening step due to low correlation or fish detection, and if enabled by the EXxxx1x command, then a three-beam solution is calculated by the ADCP. This is accomplished by replacing the bad radial beam velocity with a value calculated from the last row of the instrument transformation matrix so as to force the error velocity to zero. Indeed, the actual error velocity cannot be computed in this case, because there is no longer any redundant information. The X, Y, and Z components in the instrument coordinates are then calculated in the usual way using the first three rows of the instrument transformation matrix.

For example, let's assume our standard WorkHorse ADCP has marked only Beam4 bad:

We then need to use 3Beam Solution.

As aforementioned, since we only have 3 Beams available we cannot compute the error velocity as we do not have redundancy in the data anymore. Therefore, we can zero the error velocity:

$$e=0$$

Now if we replace the error velocity in the last row of the transformation matrix described in previous chapter, we obtain:

$$b4 = b1 + b2 - b3 \quad (3)$$

Now that we have expressed the bad Beam in function of the other Beams themselves expressed in function of Beam4, let's replace Beam4 expression above (3) in the transformation matrix described in previous chapter. We then obtain the 3Beam Solution Transformation matrix for a WorkHorse ADCP with Beam4 Bad:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1 ca - b_2 ca \\ b_1 ca + b_2 ca - 2b_3 ca \\ 2b(b_1 + b_2) \end{bmatrix} \quad (4)$$